

Propagation of Waves Through Magnetoplasma Slab Within a Parallel-Plate Guide

HUA-CHENG CHANG, SHYH-KANG JENG, RUEY-BEEI WU,
AND CHUN HSIUNG CHEN

Abstract—By applying the variational reaction theory, a variational equation is established for handling wave propagation in a parallel-plate guide within which a magnetized inhomogeneous lossy plasma slab is inserted. The equation is then solved by the finite-element method along with the frontal solution algorithm. With such an approach, the reflection coefficient and the field distribution in the slab are obtained. In this study, the factors which may influence the propagation characteristics of the guide are studied. These factors include the plasma electron density, the strength and the direction of the static magnetic field, the width and the thickness of the slab, and the electron collision losses. A special modal expansion solution is also incorporated to investigate an anomalous numerical instability associated with the present numerical algorithm.

I. INTRODUCTION

A PLASMA–DIELECTRIC sandwich structure used as a microwave filter and with voltage (and, hence, electron density) tuning has been reported recently [1]. With only layers of isotropic and homogeneous media, this sandwich structure can be easily handled by the conventional transmission-line techniques [2], [3]. However, if the slab material within the guide is anisotropic and inhomogeneous, the associated problem then becomes very complicated and is difficult to access.

For a simplified propagation problem through a one-dimensional inhomogeneous plasma slab in an unbounded region, some investigators [4], [5] made use of a special technique to figure out the governing variational equation and then used the finite-element method to find out the field distribution. The extension of this technique to the same propagation problem in the presence of an external static magnetic field was unsuccessful owing to the introduction of material anisotropy. This difficulty has been resolved by the methodology of variational electromagnetics [6], [7] and the variational reaction theory [8].

By extending the previous studies, this paper deals with the guided wave problem in which a magnetized plasma slab is placed within an infinitely extended parallel-plate metallic guide. The reason to select this parallel-plate structure is that it may reduce the mathematics involved and still allow a simplified analysis for a more practical strip

Manuscript received September 18, 1984; revised August 27, 1985. This work was supported in part by the National Science Council, Republic of China, under Grant NSC-71-0201-E002-14.

The authors are with the Department of Electrical Engineering, National Taiwan University, Taipei, Taiwan, R.O.C.

IEEE Log Number 8405925.

transmission line where a magnetized plasma slab is incorporated.

In this study, the governing variational equation is first derived, using the variational reaction theory. The equation is then solved by the finite-element method coupled with the frontal solution algorithm. In comparison with the previous works and the modal expansion solutions (Appendix), the validity of the approach can be confirmed. Finally included are the numerical results for specifying the characteristics of the particular guiding structure.

II. VARIATIONAL FORMULATION

Consider an infinitely extended parallel metallic plate guide as shown in Fig. 1 within which a slab of anisotropic, inhomogeneous, and lossy material $(\mu_0 \bar{\mu}(x, y), \epsilon_0 \bar{\epsilon}(x, y))$ is inserted. Here, $\bar{\mu}(x, y)$ and $\bar{\epsilon}(x, y)$ represent the relative permeability and permittivity tensors, respectively. Assume that the slab occupies the region $0 \leq x \leq a$, $0 \leq y \leq b$, and is illuminated by a TEM wave

$$\begin{aligned} \bar{E}_0^t &= \hat{y} E_0^t \exp(-jk_0 x) \\ \eta_0 \bar{H}_0^t &= \hat{z} E_0^t \exp(-jk_0 x) \\ k_0 &= \omega \sqrt{\mu_0 \epsilon_0} \quad \eta_0 = \sqrt{\mu_0 / \epsilon_0}. \end{aligned} \quad (1)$$

As usual, the time-harmonic factor $\exp(j\omega t)$ is adopted throughout this study.

In the slab, we shall let

$$\begin{aligned} \bar{\epsilon} &= \begin{bmatrix} \epsilon_{xx} & \epsilon_{xy} & \epsilon_{xz} \\ \epsilon_{yx} & \epsilon_{yy} & \epsilon_{yz} \\ \epsilon_{zx} & \epsilon_{zy} & \epsilon_{zz} \end{bmatrix} = \begin{bmatrix} \bar{\epsilon}_{tt} & \bar{\epsilon}_{tz} \\ \bar{\epsilon}_{zt}^T & \epsilon_{zz} \end{bmatrix} \\ \bar{\mu} &= \begin{bmatrix} \bar{\mu}_{tt} & \bar{\mu}_{tz} \\ \bar{\mu}_{zt}^T & \mu_{zz} \end{bmatrix} \end{aligned} \quad (2)$$

where the subscript t means the component transverse to the z -direction and the superscript T means the transpose of a matrix. Obviously, these two tensors are unit tensors outside the slab.

By the variational reaction theory [8], we can achieve the (E_z, H_z) formulation by choosing the longitudinal fields (E_z, H_z) as unknowns. First, we constrain the transverse sources (\bar{J}_t, \bar{M}_t) to be zero and represent the transverse

fields (\bar{E}_t, \bar{H}_t) in terms of the longitudinal ones, i.e.,

$$\begin{aligned}\bar{E}_t &= \frac{1}{j\omega\epsilon_0} \bar{\epsilon}_{tt}^{-1} \cdot (\nabla H_z \times \hat{z} - j\omega\epsilon_0 \bar{\epsilon}_{tz} E_z) \\ \bar{H}_t &= \frac{-1}{j\omega\mu_0} \bar{\mu}_{tt}^{-1} \cdot (\nabla E_z \times \hat{z} + j\omega\mu_0 \bar{\mu}_{tz} H_z).\end{aligned}\quad (3)$$

Then, we further constrain the longitudinal sources (J_z, M_z) to be zero outside the slab so that the exterior (E_z, H_z) fields are

$$\begin{aligned}E_z &= \begin{cases} \sum_{m=1}^{\infty} R_m^{TE} \sin\left(\frac{m\pi y}{b}\right) e^{jk_m x}, & x \leq 0 \\ \sum_{m=1}^{\infty} T_m^{TE} \sin\left(\frac{m\pi y}{b}\right) e^{jk_m(a-x)}, & x \geq a \end{cases} \\ \eta_0 H_z &= \begin{cases} E_0^i e^{-jk_0 x} - \sum_{m=0}^{\infty} R_m^{TM} \cos\left(\frac{m\pi y}{b}\right) e^{jk_m x}, & x \leq 0 \\ \sum_{m=0}^{\infty} T_m^{TM} \cos\left(\frac{m\pi y}{b}\right) e^{jk_m(a-x)}, & x \geq a \end{cases}\end{aligned}\quad (4)$$

where R and T are the modal coefficients to be determined and

$$k_m^2 = k_0^2 - (m\pi/b)^2, \quad m = 1, 2, \dots \quad (5)$$

The exterior (\bar{E}_t, \bar{H}_t) fields can be expressed easily in terms of the boundary (E_z, H_z) fields by (3). Also, these modal coefficients can be expressed in terms of boundary fields by matching the continuity conditions, i.e.,

$$\begin{aligned}R_0^{TM} &= \frac{1}{b} \int_0^b [E_0^i - \eta_0 H_z(x=0)] dy \\ T_0^{TM} &= \frac{1}{b} \int_0^b \eta_0 H_z(x=a) dy \\ R_m^{TE} &= \frac{2}{b} \int_0^b E_z(x=0) \sin\left(\frac{m\pi y}{b}\right) dy, \quad m = 1, 2, \dots \\ T_m^{TE} &= \frac{2}{b} \int_0^b E_z(x=a) \sin\left(\frac{m\pi y}{b}\right) dy \\ R_m^{TM} &= \frac{-2}{b} \int_0^b \eta_0 H_z(x=0) \cos\left(\frac{m\pi y}{b}\right) dy \\ T_m^{TM} &= \frac{2}{b} \int_0^b \eta_0 H_z(x=a) \cos\left(\frac{m\pi y}{b}\right) dy.\end{aligned}\quad (6)$$

Now the variational formula becomes

$$\begin{aligned}\delta I &= 0 \\ I &= \int_0^a \int_0^b (E_z J_z - H_z M_z) dy dx\end{aligned}\quad (7)$$

where

$$\begin{aligned}J_z &= \hat{z} \cdot (\nabla \times \bar{H} - j\omega\epsilon_0 \bar{\epsilon} \cdot \bar{E}) \\ M_z &= \hat{z} \cdot (-\nabla \times \bar{E} - j\omega\mu_0 \bar{\mu} \cdot \bar{H}).\end{aligned}$$

By substituting (3)–(6) into (7) and making certain simplifications, we finally obtain the variational formula for

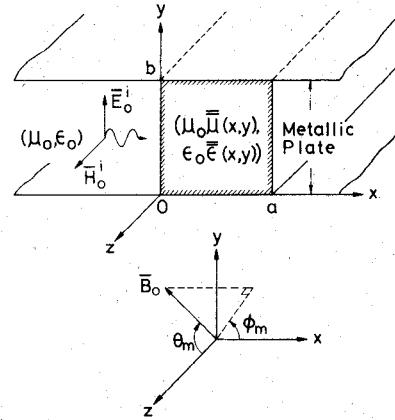


Fig. 1. Parallel-plate guide with magnetized, inhomogeneous, and lossy plasma slab.

the unknown (E_z, H_z) fields as follows:

$$\begin{aligned}I &= \int_0^a \int_0^b [j\omega\epsilon_0 (\bar{E}_t^a \cdot \bar{\epsilon}_{tt} \cdot \bar{E}_t - E_z^a \epsilon_{zz} E_z) \\ &\quad - j\omega\mu_0 (\bar{H}_t^a \cdot \bar{\mu}_{tt} \cdot \bar{H}_t - H_z^a \mu_{zz} H_z)] dy dx \\ &\quad + \left[\sum_{m=1}^{\infty} (\eta_0 k_m / k_0) \frac{2}{b} \int_0^b H_z^a \cos\left(\frac{m\pi y}{b}\right) dy \right. \\ &\quad \cdot \left. \int_0^b H_z \cos\left(\frac{m\pi y}{b}\right) dy \right]_{x=0, a} \\ &\quad - \left[\sum_{m=1}^{\infty} (k_m / \eta_0 k_0) \frac{2}{b} \int_0^b E_z^a \sin\left(\frac{m\pi y}{b}\right) dy \right. \\ &\quad \cdot \left. \int_0^b E_z \sin\left(\frac{m\pi y}{b}\right) dy \right]_{x=0, a} \\ &\quad + \frac{1}{b} \int_0^b H_z^a(x=0) dy \cdot \int_0^b (\eta_0 H_z(x=0) - 2E_0^i) dy.\end{aligned}\quad (8)$$

Here, $(\bar{E}_t^a, \bar{H}_t^a)$ are related to (E_z^a, H_z^a) by

$$\begin{aligned}\bar{E}_t^a &= \frac{1}{j\omega\epsilon_0} (\bar{\epsilon}_{tt}^T)^{-1} \cdot (\nabla H_z^a \times \hat{z} - j\omega\epsilon_0 \bar{\epsilon}_{tz} E_z^a) \\ \bar{H}_t^a &= \frac{-1}{j\omega\mu_0} (\bar{\mu}_{tt}^T)^{-1} \cdot (\nabla E_z^a \times \hat{z} + j\omega\mu_0 \bar{\mu}_{tz} H_z^a).\end{aligned}\quad (9)$$

III. FINITE-ELEMENT COMPUTATION AND ITS VALIDATION

The resultant variational equation (8) will be solved by the finite-element technique [7], [9], [10]. First, the whole region $0 \leq x \leq a$, $0 \leq y \leq b$ is divided into $10a/\lambda_0 \times 10b/\lambda_0 \sim 20a/\lambda_0 \times 20b/\lambda_0$ elements, the boundaries $x = 0$ and $x = a$ are considered as two elements, and in each element several sampling nodes are chosen. The fields in each element are interpolated, using the isoparametric quadrature basis functions [9]. Next, the Ritz procedure [7], [11] is employed to obtain a linear matrix equation. This equation, then, is solved through a frontal solution algorithm [7] which assembles and solves the final matrix

equation element by element. This algorithm is rather efficient in saving a computer's core memory.

The problem that we are dealing with is the propagation of waves in a magnetized cold plasma ($\mu_0 \bar{\mu}, \epsilon_0 \bar{\epsilon}$); thereby,

$$\bar{\mu} = \bar{\epsilon} = \bar{1}$$

$$\bar{\epsilon} = \bar{1} - X_N \cdot \begin{bmatrix} U & jY_H l_z & -jY_H l_y \\ -jY_H l_z & U & jY_H l_x \\ jY_H l_y & -jY_H l_x & U \end{bmatrix}^{-1}$$

$$U = 1 - jZ_c$$

$$l_x = \sin \theta_m \cos \phi_m \quad l_y = \sin \theta_m \sin \phi_m \quad l_z = \cos \theta_m. \quad (10)$$

Here the parameters X_N , Y_H , and Z_c are proportional to the electron density N_e , the strength of the static magnetic field B_0 , and the average collision frequency of electrons ν , respectively, [12]:

$$X_N = \frac{\omega_N^2}{\omega^2} \quad \omega_N^2 = \frac{N_e q_e^2}{m_e \epsilon_0} \quad Y_H = \frac{\omega_H}{\omega}$$

$$\omega_H = \frac{q_e B_0}{m_e} \quad Z_c = \frac{\nu}{\omega}. \quad (11)$$

As usual, q_e and m_e represent the charge and mass of an electron. The parameters θ_m and ϕ_m are the azimuthal and polar angles of the static magnetic field (Fig. 1).

In a computer simulation, we assume that Y_H , θ_m , ϕ_m , and Z_c are constant. Only two electron density profiles are considered in this paper, namely, the homogeneous one

$$X_N = X_m, \quad 0 \leq x \leq a, \quad 0 \leq y \leq b \quad (12)$$

and the parabolic one

$$X_N = X_m [1 - (2x/a - 1)^2] \cdot [1 - (2y/b - 1)^2],$$

$$0 \leq x \leq a, \quad 0 \leq y \leq b. \quad (13)$$

If necessary, the strength of the static magnetic field and the losses may also be inhomogeneous since the present approach can handle the problems without increasing the complexity.

To ensure the validity of this approach and the associated computer program, we first consider the case with a lossless nonmagnetized homogenous profile defined by (12). Actually, this case has been solved exactly by the propagation matrix method [3] in a recent study [1]. Fig. 2 shows excellent agreement between their results and the present ones (indicated by the symbol Δ). It provides at least evidence of confirming the validity of the program. The second check is made on the study of a nonmagnetized plasma-dielectric sandwich filter [1]. Both the previous study and our approach lead to the same center frequency and bandwidth for any transmission window. The third verification is conducted on the lossless magnetized plasma with $\bar{B}_0 \parallel \bar{E}_0'$ and with profile (12), from which Fig. 2 is again obtained. In this case, the Lorentz force [13] cannot affect the electron's motion; thus, the behavior of the wave should be the same whether B_0 is zero or not.

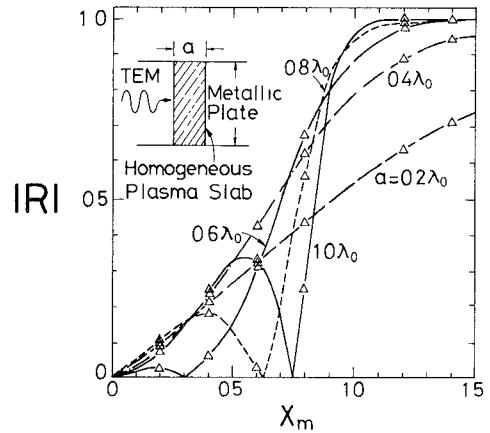


Fig. 2. Curves of reflection coefficient versus X_m to check validity of our approach. Homogeneous plasma slab is adopted with parameters $Z_c = 0$ and $\bar{B}_0 = 0$ or $\bar{B}_0 = \hat{y} B_0$ (i.e., $\bar{B}_0 \parallel \bar{E}_0'$). λ_0 is the free-space wavelength.

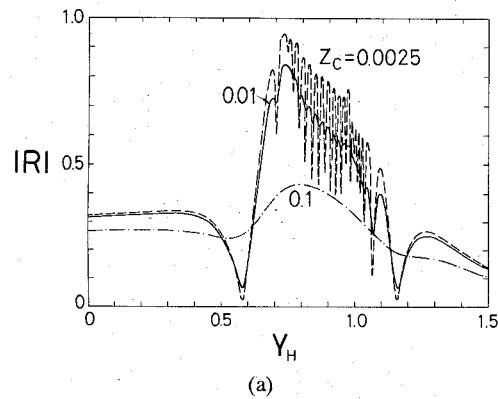
IV. NUMERICAL RESULTS AND DISCUSSIONS

Numerical results to characterize the guiding system in Fig. 1 are presented in this section. Here we assume that a TEM wave is incident upon the plasma slab and that the parallel-plate guide without the slab can only transmit the TEM wave, thus leaving all other modes to be below cutoff. Specifically, the reflection coefficient associated with the TEM wave, $R = R_0^{\text{TM}}$, will be computed and examined. Shown in Figs. 3-7 are amplitudes of reflection coefficient versus Y_H with Z_c , b , and X_m (the maximum of X_N) as parameters.

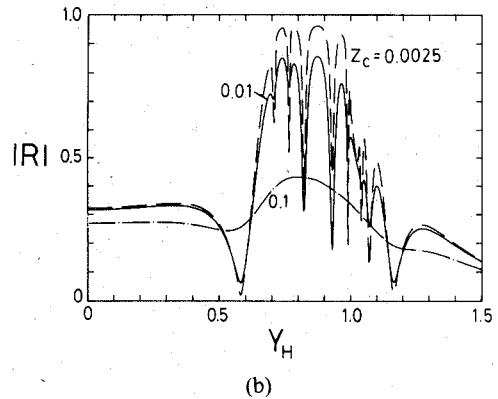
In Fig. 3(a), where the homogeneous profile (12) with $X_m = 0.5$ is considered, it is found that all curves of $|R|$ undergo steady change when Y_H is small, and decrease smoothly when Y_H is large. However, a large and rapid undulation is observed in the range $0.7 < Y_H < 1.06$ as losses are very small ($Z_c = 0.0025$), and the undulation phenomenon diminishes gradually ($Z_c = 0.01$) and finally disappears ($Z_c = 0.1$) as losses are increased.

To probe the source of such a mysterious phenomenon, we have to solve exactly some specific problems. Fortunately, in Fig. 3, where we assumed $\bar{B}_0 = B_0 \hat{z}$, we may get a simpler $\bar{\epsilon}_{rr}$. This enables us to have a modal expansion solution (see the Appendix for a brief discussion). The modal expansion results are exhibited in Fig. 3(b), and we can notice easily that the curves are almost the same except in the undulation interval of Fig. 3(a). This indicates that the range $0.7 < Y_H < 1.06$ is a difficult region for our approach. As depicted in the Appendix, this discrepancy might arise partly because the fundamental mode in the slab becomes evanescent as $1 > Y_H > \sqrt{1 - X_N} \sim 0.7$, and partly because this mode is drastically oscillatory as $Y_H \geq 1$. When Z_c increases, the error—and the undulation—becomes negligible.

The effects of varying X_m , b , and Z_c are also examined. From Figs. 4-6, we present the computed results and their modal expansion counterparts. A similar undulation again appears when b and X_m are not too small, and also regrettably disappears in the model expansion results.



(a)



(b)

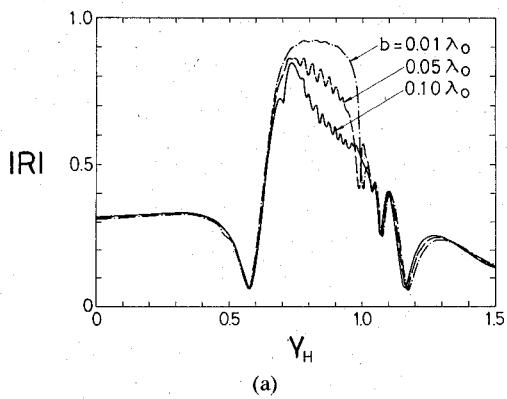
Fig. 3. Amplitudes of reflection coefficient versus magnetization factor Y_H with losses Z_c as parameters. Homogeneous profile (12) is considered with guide parameters: $a = \lambda_0$, $b = 0.1\lambda_0$, $X_m = 0.5$, and $\bar{B}_0 = 2B_0$ (i.e., $\bar{B}_0 \perp \bar{E}_0$ and \bar{k}_0). (a) Variational solutions. (b) Modal expansion solutions.

Finally, we inspect the parabolic profile (13) cases, in which the modal expansion solution happens to be unavailable. The computed results together with a homogeneous profile case for comparison are displayed in Fig. 7. It seems that the undulation is more insignificant. Thus, we may guess that the present approach can do better for the more realistic parabolic modeling of plasma electron density.

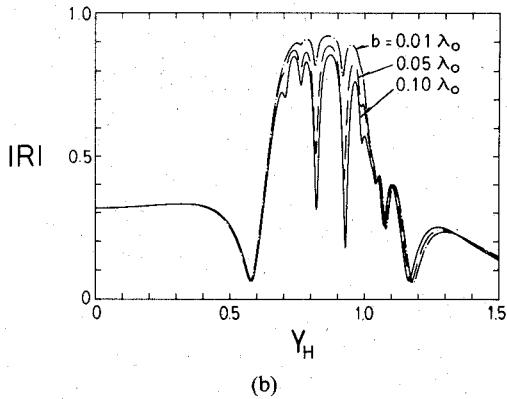
V. CONCLUSIONS

Based on the variational reaction theory, a variational equation has been established for handling the problem of wave propagation through a magnetized inhomogeneous plasma slab which is within a parallel-plate guide. The equation has been solved and converted into a computer program, using the techniques of finite-element and frontal solutions. The computer program has also been validated by comparing its results for several specific cases to the ones of previous studies.

This study also reveals some vulnerable spots of this approach in dealing with the “undulation” region. However, it is also verified that this approach is reliable when the parallel plates are closer, the electron density is lower, and the loss is larger. A more solid theoretical analysis to handle this numerical instability is worthy of further investigation.

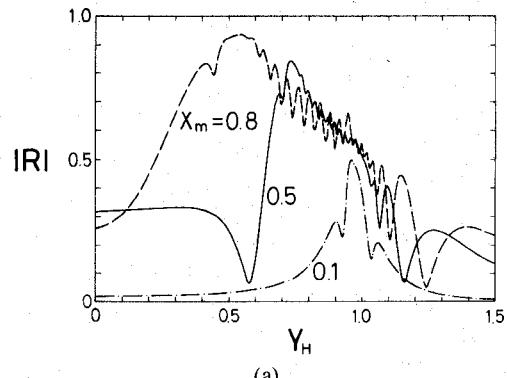


(a)

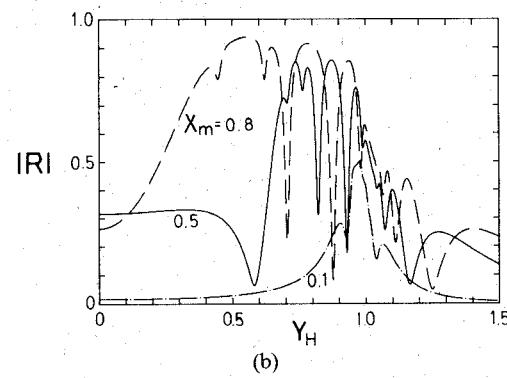


(b)

Fig. 4. Reflection coefficients versus Y_H with guide height b as parameters. Homogeneous profile is adopted with parameters: $a = \lambda_0$, $X_m = 0.5$, $Z_c = 0.01$, and $\bar{B}_0 = 2B_0$. (a) Variational solutions. (b) Modal expansion solutions.



(a)



(b)

Fig. 5. Reflection coefficients versus Y_H with X_m as parameters. Homogeneous profile is adopted with parameters: $a = \lambda_0$, $b = 0.1\lambda_0$, $Z_c = 0.01$, and $\bar{B}_0 = 2B_0$. (a) Variational solutions. (b) Modal expansion solutions.

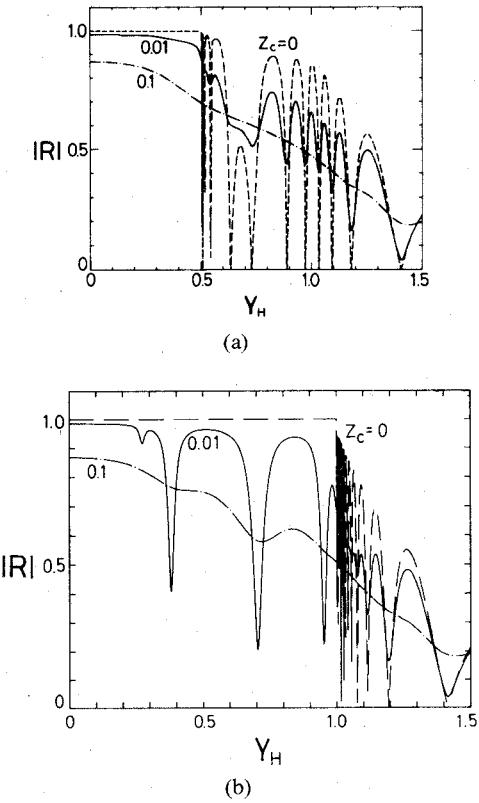


Fig. 6. Computed results for $X_m = 1.5$ with Y_H as variables and Z_c as parameters. Plasma slab of homogeneous profile is adopted with $a = \lambda_0$, $b = 0.1\lambda_0$, and $\bar{B}_0 = \bar{B}_0$. (a) Variational solutions. (b) Modal expansion solutions.

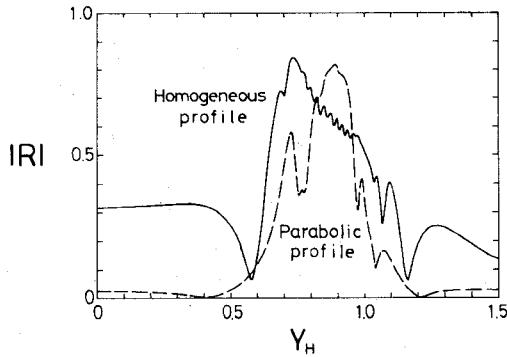


Fig. 7. Computed results for homogeneous profile (12) and parabolic profile (13) to illustrate the effect of profile inhomogeneity. Associated guide parameters are $a = \lambda_0$, $b = 0.1\lambda_0$, $X_m = 0.5$, $Z_c = 0.01$, and $\bar{B}_0 = \bar{B}_0$.

APPENDIX MODAL EXPANSION METHOD

Once we can find the modal solutions in the magneto-plasma slab sandwiched by the parallel-plate guide, we can apply the modal expansion method to solve the wave-propagation problem as shown in Fig. 1. When the plasma slab is magnetized along the z -direction, the relative permittivity tensor is

$$\bar{\epsilon} = \begin{bmatrix} \epsilon_1 & j\epsilon_2 & 0 \\ -j\epsilon_2 & \epsilon_1 & 0 \\ 0 & 0 & \epsilon_{zz} \end{bmatrix} \quad (A1)$$

where

$$\begin{aligned} \epsilon_1 &= 1 - UX_N/(U^2 - Y_H^2) \\ \epsilon_2 &= Y_H X_N/(U^2 - Y_H^2) \\ \epsilon_{zz} &= 1 - X_N/U. \end{aligned}$$

For the incident TEM wave as shown in Fig. 1, only the extraordinary wave will be excited in the slab. The modal solutions are thus

$$\begin{aligned} E_x &= \frac{jk_0 b}{m\pi} \left(1 - \frac{k_{xm}^2}{k_0^2 \epsilon_1} \right) \sin \frac{m\pi y}{b} e^{-jk_{xm} x} \\ \eta_0 H_z &= \left(\cos \frac{m\pi y}{b} - \frac{bk_{xm} \epsilon_2}{m\pi \epsilon_1} \sin \frac{m\pi y}{b} \right) e^{-jk_{xm} x} \\ E_y &= \left(\frac{k_{xm}}{k_0 \epsilon_1} \cos \frac{m\pi y}{b} - \frac{bk_0 \epsilon_2}{m\pi \epsilon_1} \sin \frac{m\pi y}{b} \right) e^{-jk_{xm} x} \\ k_{xm}^2 &= k_0^2 (\epsilon_1 - \epsilon_2^2 / \epsilon_1) - (m\pi/b)^2, \quad m = 1, 2, \dots \end{aligned} \quad (A2)$$

Peculiarly, the fundamental mode cannot be achieved from (A2) with $m = 0$ if only $\epsilon_2 \neq 0$. However, the mode is still achievable with $k_{x0}^2 = k_0^2 \epsilon_1$ instead; i.e.,

$$\begin{aligned} E_x &= 0 \\ \eta_0 H_z &= \exp [k_{x0} (-\epsilon_2 y / \epsilon_1 - jx)] \\ E_y &= (k_0 / k_{x0}) \cdot \eta_0 H_z. \end{aligned} \quad (A3)$$

For the lossless cases, this mode becomes evanescent in the x -direction as $\epsilon_1 < 0$, i.e.,

$$1 > Y_H^2 > 1 - X_N. \quad (A4)$$

ACKNOWLEDGMENT

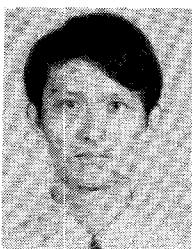
Discussions with Prof. Y. Y. Sun, Prof. W. S. Wang, Dr. Y. W. Kiang, Dr. P. Hsu, and Dr. H. C. Chang were helpful and are much appreciated.

REFERENCES

- [1] G. C. Tai, C. H. Chen, and Y. W. Kiang, "Plasma-dielectric sandwich structure used as a tunable bandpass microwave filter," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-32, pp. 111-113, Jan. 1984.
- [2] L. M. Magid, *Electromagnetic Fields, Energy, and Waves*. New York: Wiley, 1972.
- [3] J. A. Kong, *Theory of Electromagnetic Waves*. New York: Wiley, 1975.
- [4] C. H. Chen and C. D. Lien, "A finite element solution of the wave propagation problem for an inhomogeneous dielectric slab," *IEEE Trans. Antennas Propagat.*, vol. AP-27, pp. 877-880, Nov. 1979.
- [5] C. H. Chen and Y. W. Kiang, "A variational theory for wave propagation in a one-dimensional inhomogeneous medium," *IEEE Trans. Antennas Propagat.*, vol. AP-28, pp. 762-769, Nov. 1980.
- [6] S. K. Jeng and C. H. Chen, "Variational finite element solution of electromagnetic wave propagation in a one-dimensional inhomogeneous anisotropic medium," *J. Appl. Phys.*, vol. 55, pp. 630-636, Feb. 1984.
- [7] S. K. Jeng and C. H. Chen, "On variational electromagnetics: theory and application," *IEEE Trans. Antennas Propagat.*, vol. AP-32, pp. 902-907, Sept. 1984.
- [8] R. B. Wu and C. H. Chen, "On the variational reaction theory for dielectric waveguides," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-33, pp. 477-483, June 1985.

- [9] E. Hinton and D. R. J. Owen, *Finite Element Programming*. New York: Academic Press, 1977.
- [10] D. H. Norrie and G. de Vries, *An Introduction to Finite Element Analysis*. New York: Academic Press, 1978.
- [11] C. H. Chen and C. D. Lien, "The variational principle for non-self-adjoint electromagnetic problems," *IEEE Trans. Microwave Theory Tech.*, vol. MTT-28, pp. 878-886, Aug. 1980.
- [12] K. G. Budden, *Radio Waves in the Ionosphere*. London: Cambridge University Press, 1961.
- [13] C. M. Huang, *Wave Propagation*. Taipei: Lien-Ching, 1978.

†



Hua-Cheng Chang was born in I-Lan, Taiwan, Republic of China, on May 21, 1959. He received the B.S.E.E. and M.S.E.E. degrees from National Taiwan University, Taipei, Taiwan, in 1981 and 1983, respectively. He is now in the Ph.D. program at the Department of Electrical Engineering and Computer Sciences, University of California, Berkeley.

†



Shyh-Kang Jeng was born in I-Lan, Taiwan, Republic of China, on May 6, 1957. He received the B.S.E.E. and Ph.D. degrees from National Taiwan University, Taipei, Taiwan, in 1979 and 1983, respectively.

In 1981, he joined the faculty of the Department of Electrical Engineering, National Taiwan University, where he is now an Associate Professor. His topics of interest include antennas, wave propagation, and numerical technique in electromagnetics.



Ruey-Beei Wu was born in Tainan, Taiwan, Republic of China, on October 27, 1957. He received the B.S.E.E. degree from National Taiwan University, Taipei, Taiwan, in 1979, and the Ph.D. degree from the same university in 1985.

In 1982, he joined the faculty of the Department of Electrical Engineering, National Taiwan University, where he is now an Associate Professor. He is currently engaged in the area of numerical methods to electromagnetic field problems.

†



Chun Hsiung Chen was born in Taipei, Taiwan, Republic of China, on March 7, 1937. He received the B.S.E.E. degree from National Taiwan University, Taipei, Taiwan, in 1960, the M.S.E.E. degree from National Chiao Tung University, Hsinchu, Taiwan, in 1962, and the Ph.D. degree in electrical engineering from National Taiwan University in 1972.

In 1963, he joined the faculty of the Department of Electrical Engineering, National Taiwan University, where he is now a Professor. From August 1982 to July 1985, he had also been the Chairman of the same department. In 1974, he was a Visiting Researcher for one year at the Department of Electrical Engineering and Computer Sciences, University of California, Berkeley. His areas of interest include antenna and waveguide analysis, propagation and scattering of waves, and numerical techniques in electromagnetics.